



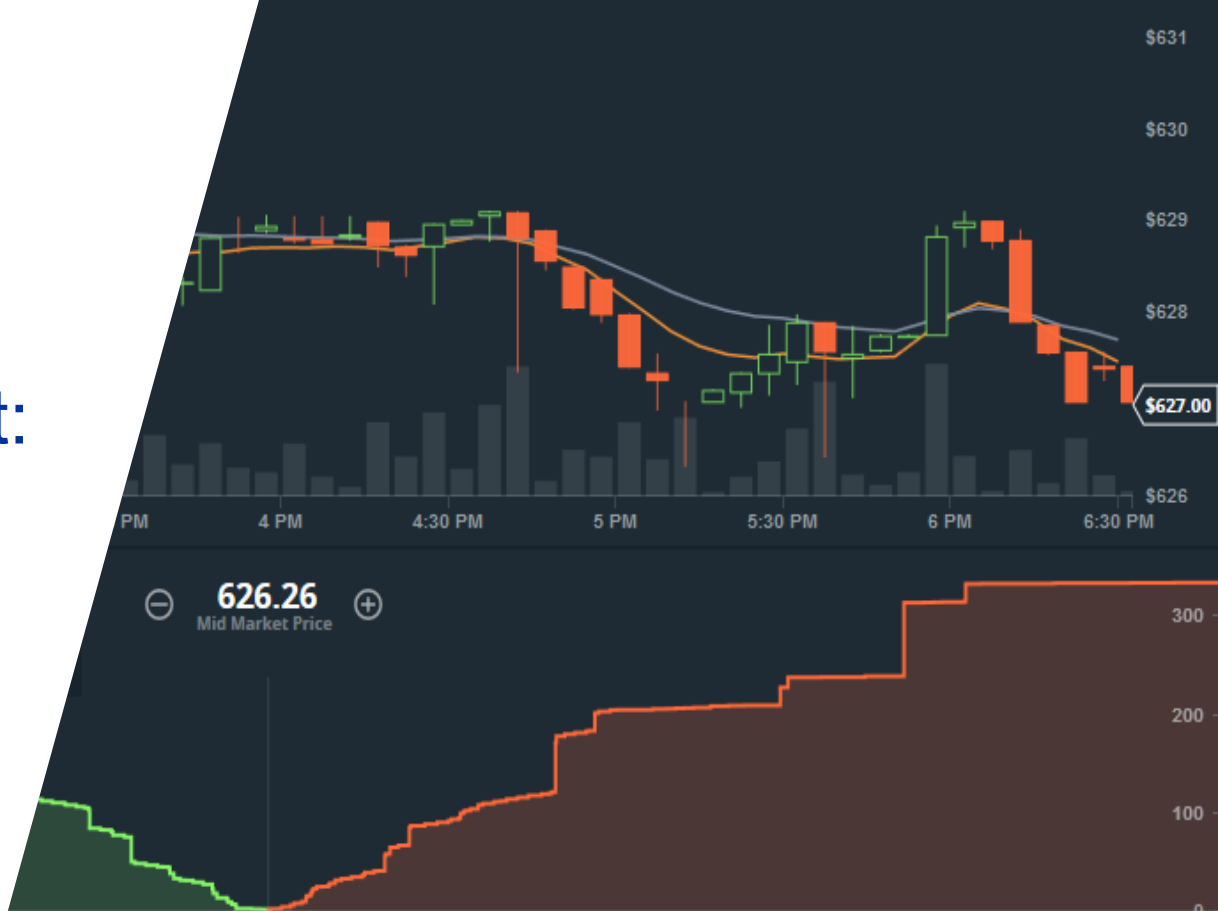
EUROPEAN CENTRAL BANK

EUROSYSTEM

# Contagion from market price impact: a price-at-risk perspective

2022 RiskLab/BoF/ESRB  
Conference on Systemic  
Risk Analytics

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# Overview

- 1 Introduction
- 2 Modelling Foundations
- 3 Data
- 4 Results and applications
- 5 Conclusion and next steps

# 1



## Introduction

# Why do we care?

Agents' **overlapping portfolios** can provide a channel of *contagion*

The **risk** stemming from this channel cannot be taken into account by any counterparty in the system: the regulator can capture the full picture

In crisis situations, modelling of **asset deleveraging** requires a *notion* of **price impact**

# Why is there price impact?

## Three approaches

Agents **successfully forecast** short term price movements and trade accordingly;

**Private information** causes trades, which cause other agents to update their valuations, thus leading to a price change;

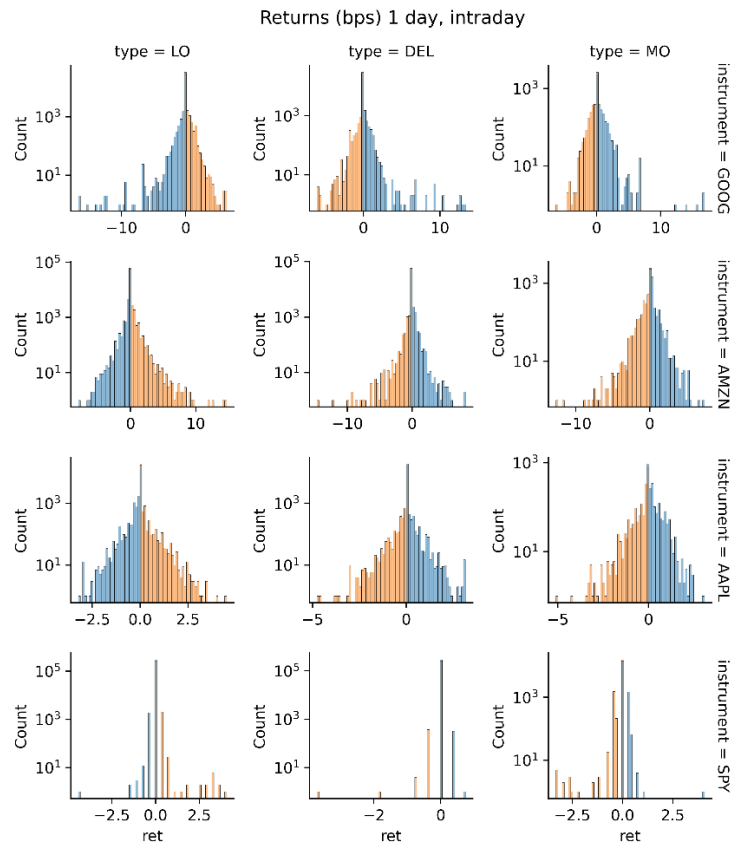
**Statistical effects** due to order flow fluctuations.

# What is price impact?

## Microstructure order book events which move the price of a security:

- Limit orders
- Limit order cancellation (deletion)
- Market orders

Note: Execution of a sell (buy) limit order corresponds to a buyer (seller) initiated trade, i.e. buy (sell) trade.



Source: LOBSTER academic sample data.

# 2



## Modelling Foundations

# How to model price impact?

## Kyle's model

$$p_t = p_{t-1} + \lambda v, \quad \text{giving } R(T) = \lambda E[v]$$

Where  $v$  is the change in observed volumes and  $\lambda$  is known as *Kyle's Lambda* and it represents the Impact parameter. The linear representation of the impact model provides a **good estimation for small volumes**, while overestimating for larger volumes

## Metaorder square-root model

$$R(T, v) \propto \sigma_T \left( \frac{Q}{V_T} \right)^\delta$$

Where  $Q$  is the total volume of the metaorder,  $\sigma_T$  is the volatility,  $V_T$  is the total market volume traded at time  $T$ .



# How to model price impact?

**Exponential specification** to impose the sublinear relationship of the volume on price impact

$$R(v) = B_{\mu} \left( 1 - \exp \left( - \frac{v * \lambda}{B_{\mu}} \right) \right)$$

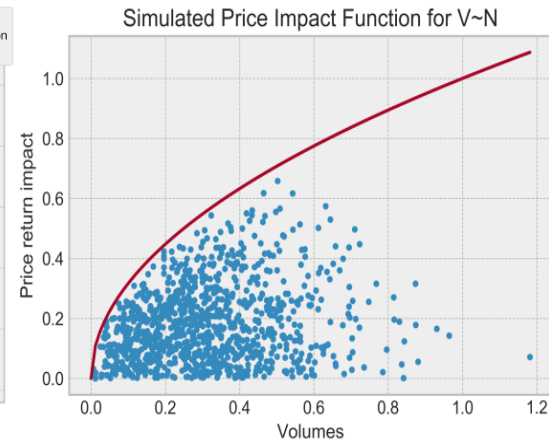
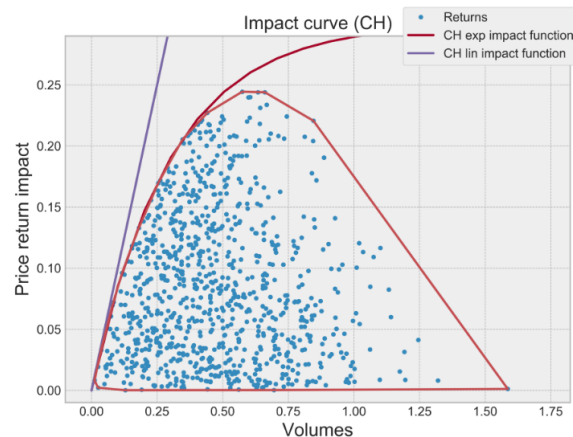
To avoid the price to drop arbitrarily close to zero, a **boundary**  $B_{\mu} > 0$  can be introduced. Consistent with the Kyle model for small volumes.

# How to model price impact?

## Convex Hull

Convex Hull (CH) of a set of points is the smallest convex subset that contains the points

- Convex hull captures returns that correspond to the more extreme buy/sell imbalances.
- Considering that prices get moved in both directions when buy and sell pressure is present, one may observe returns near zero
- Simulated data illustrates the effect of applying square-root function to aggregated daily volumes, causing impacts > 100%



Convex hull captures exactly the maximum impact at each volume level

# How to model price impact?

## Quantile regression

Expanding the exponential model: calibrate a wider range of impact severities levels, while keeping the converging nature of the exponential function.

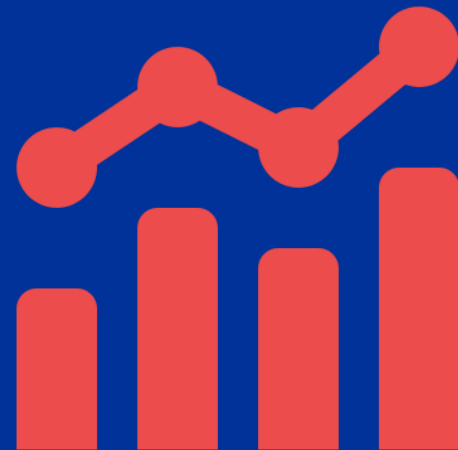
$$R(v)^q = \beta_0^q (1 - \exp(-sV)) + \beta_1^q R_{sys}$$

where  $s = \frac{\lambda}{\beta_0}$ , from which we can derive  $\lambda$ , and  $q$  is the estimated quantile.

Furthermore, a **system-level component**  $R_{sys}$  has been introduced to account for price changes due to changes in the market.

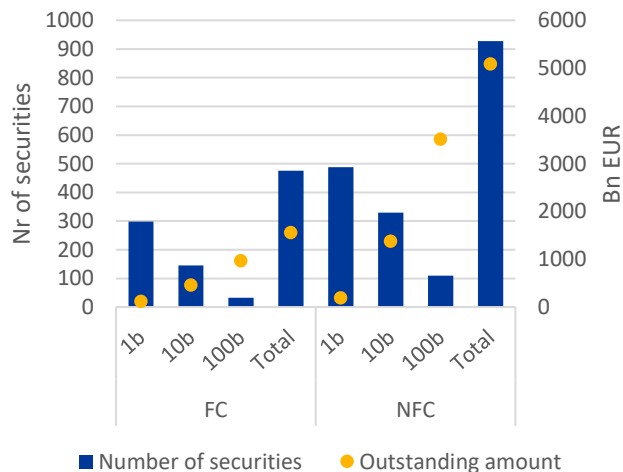
# 3

## Data

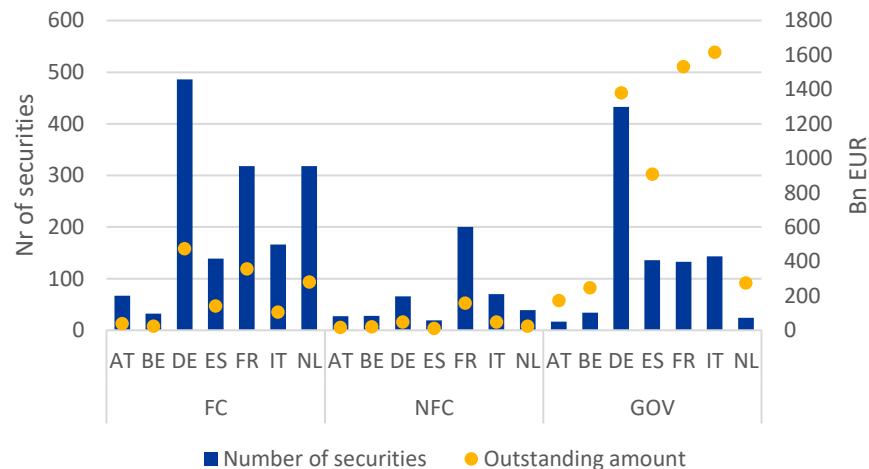


# Data coverage

## Equity security coverage



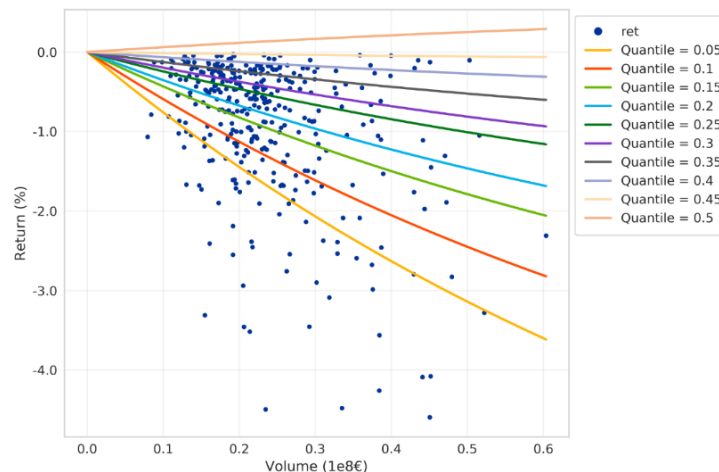
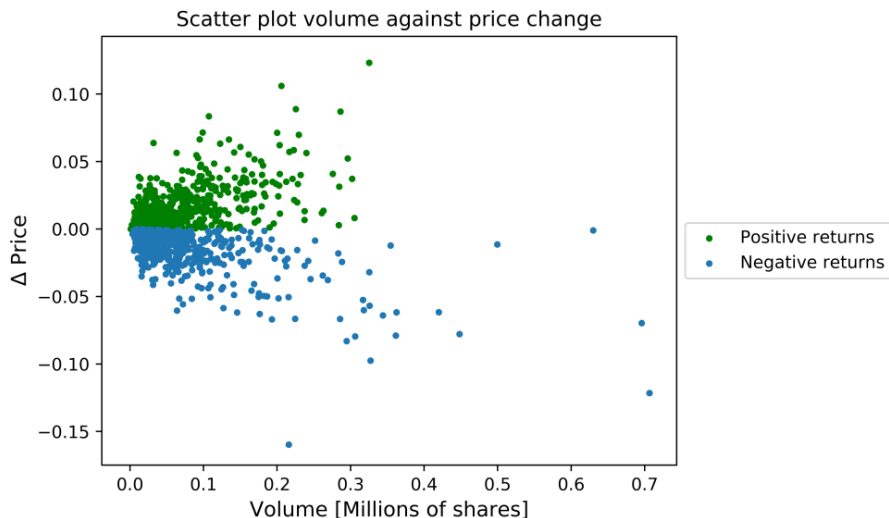
## Bond security coverage



The dataset contains an as wide as possible range of equities and bonds, including different sectors, sizes, and euro area countries amounting to 7 trillion euro equities and 7.9 trillion euro bonds.

# Visualization of price calibration on empirical data

Empirical data shows that returns diverge as volumes increase (left hand side).  
The quantile regression approach on the negative impacts allows to evaluate risk at different intensity levels (right hand side)



Source: Refinitive (Eikon)

# 4

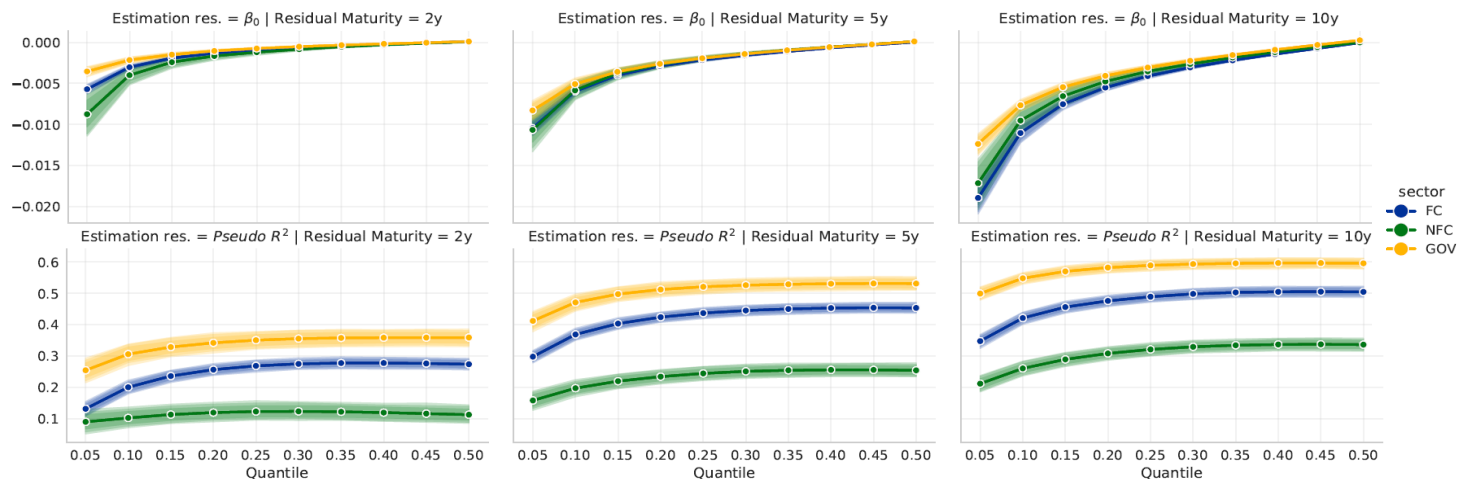


## Results and applications

# Estimation results

## Bonds

**Positive correlation** between market impact and residual maturity



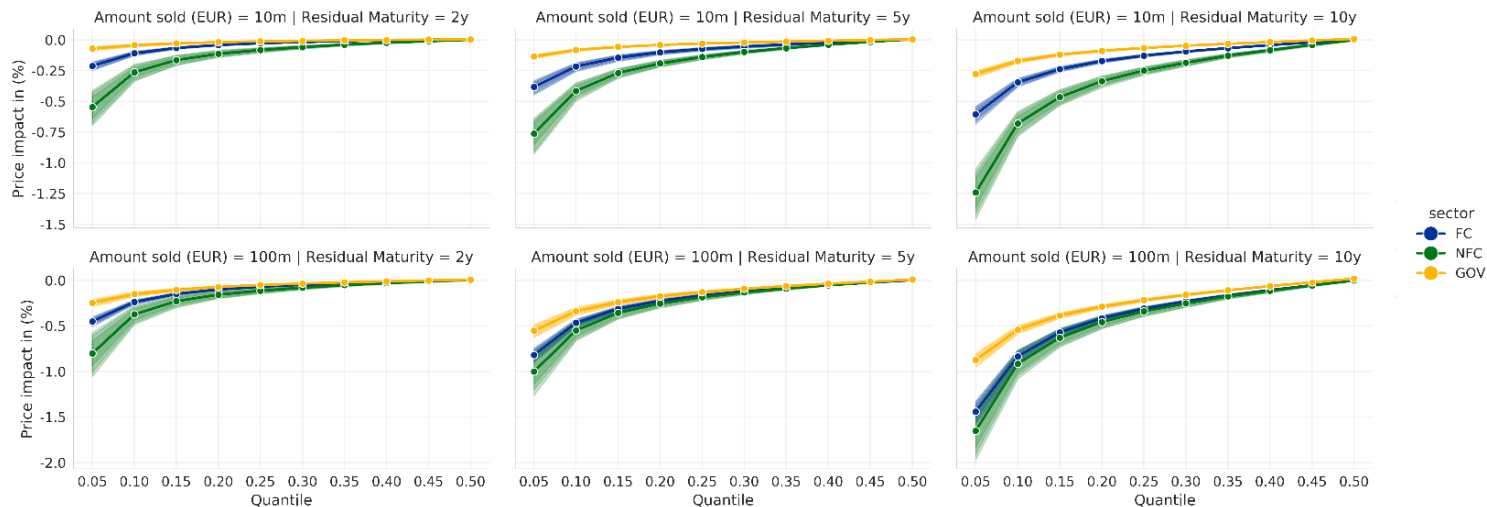
Government bonds show to be least impacted, proving their relatively low risk level.



# Security level impact size

## Bonds

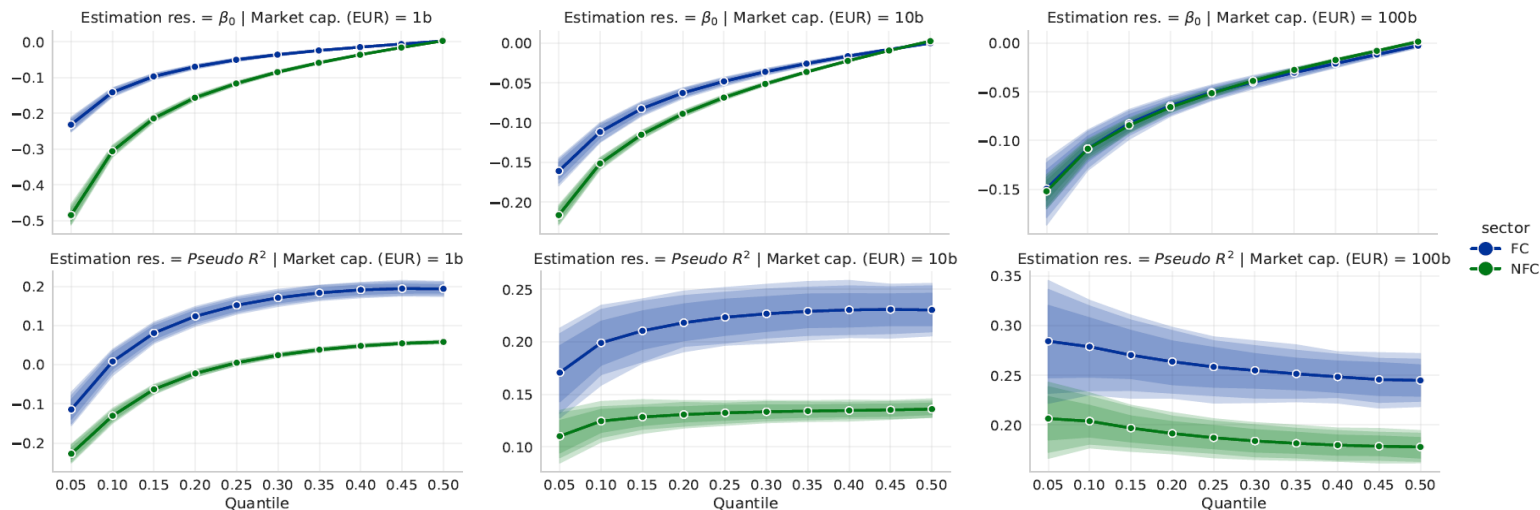
Direct market price impact from fire sale of individual bonds.



# Estimation results

## Equities

Compared to bonds, equity impact are several times more impactful.

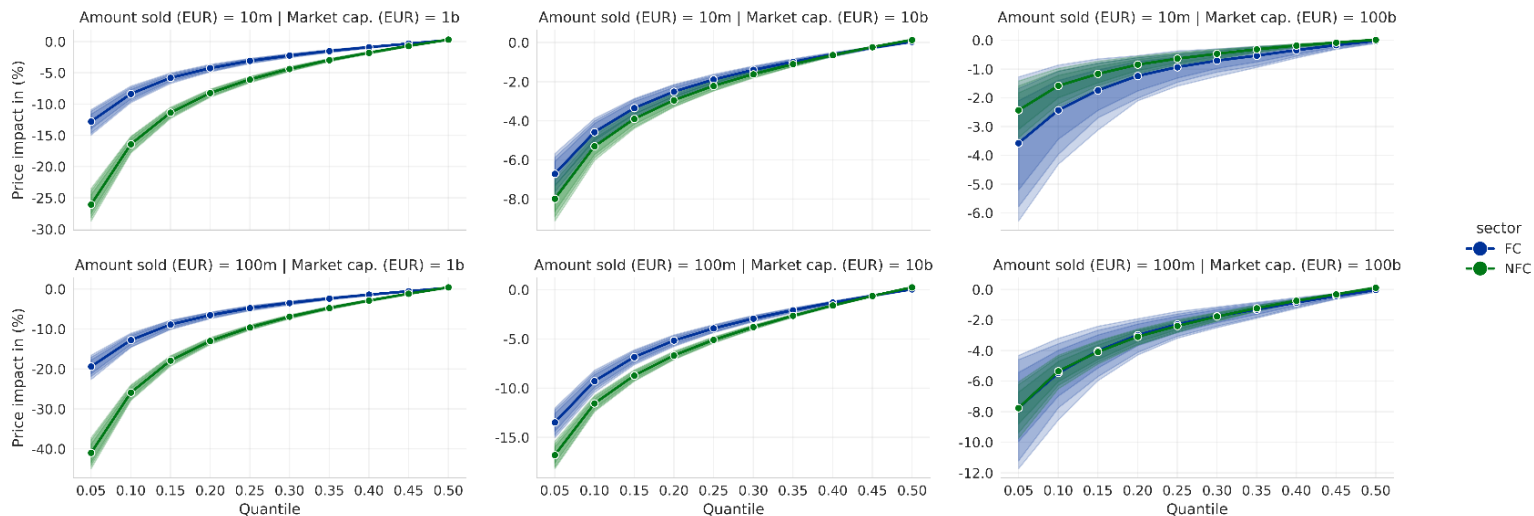


Moreover, large companies tend to absorb shocks much more than smaller companies

# Security level impact size

## Equities

Direct market price impact from fire sale of individual equity showing losses up to -40% for small cap securities

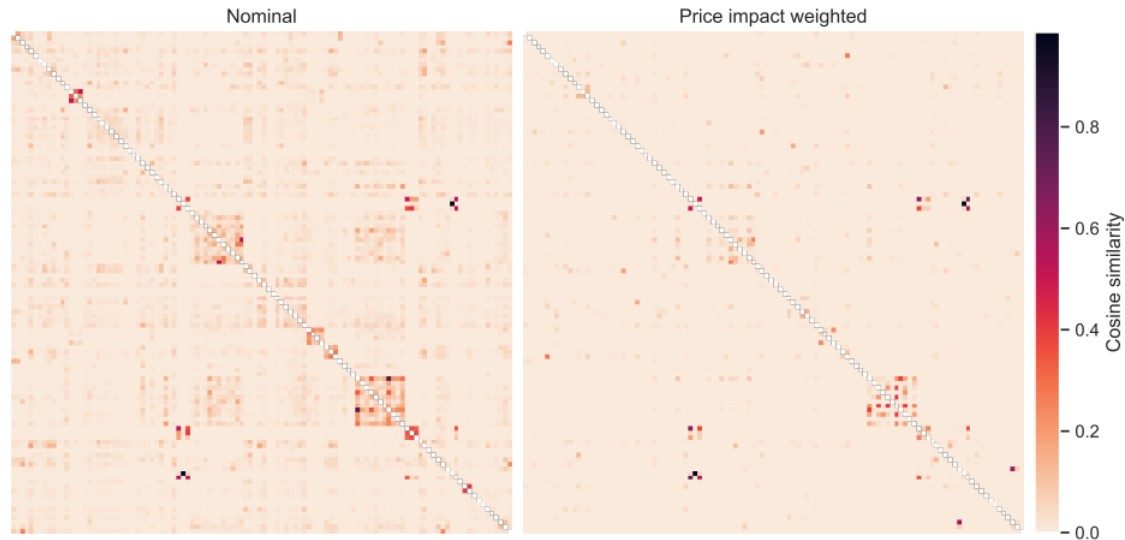


# Cosine similarity bank portfolios:

## the role of price impacts

Cosine similarity matrix (left hand side) is the degree of similarity between the exposures of two banks. In combination with individual security price impact estimations the similarity can be adjusted by heterogeneous market risk (right hand side).

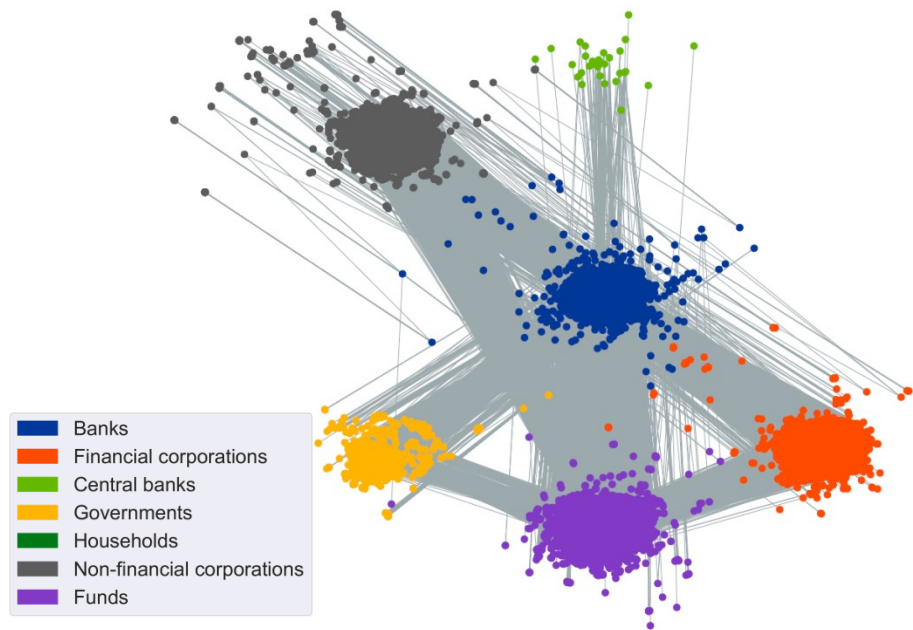
The risk weighted portfolio similarities show **the ability to lower contagion risk** by combining less risky securities such as government bonds.



# Fire sale simulations (I)

- **Using SWST model** (Sydow et al., 2021) for the system of banks and investment funds
- **Driven by liquidity shortfalls:** banks/funds cover their liquidity shortfalls by selling their tradable assets
- **Pro rata approach:** amounts sold are proportional for all securities held
- **Price equilibrium:** price impacts recalculated until no further change in market values of holdings

Securities holdings

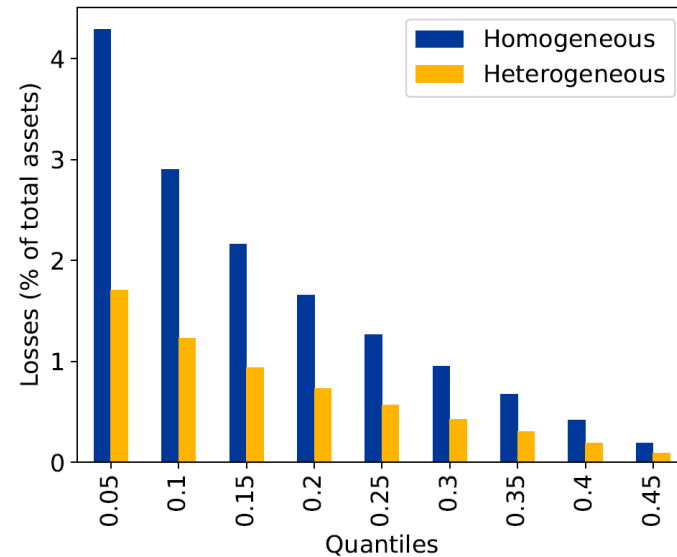


An edge shows that a bank/fund holds assets issued by another entity in a given sector. Granular securities data are covering 7% of total bank assets.

# Fire sale simulations (II)

- **Redemption shock** for investment funds to trigger fire sales of all securities in their portfolios
- **Banks and funds** suffer fire sale losses upon endogenous price drops
- Fire sale losses **largely depend** on the applied price impact parameters
- Heterogeneous impact parameters reveal **more limited risks** as opposed to homogeneous parameters

Comparison to homogeneous price impacts

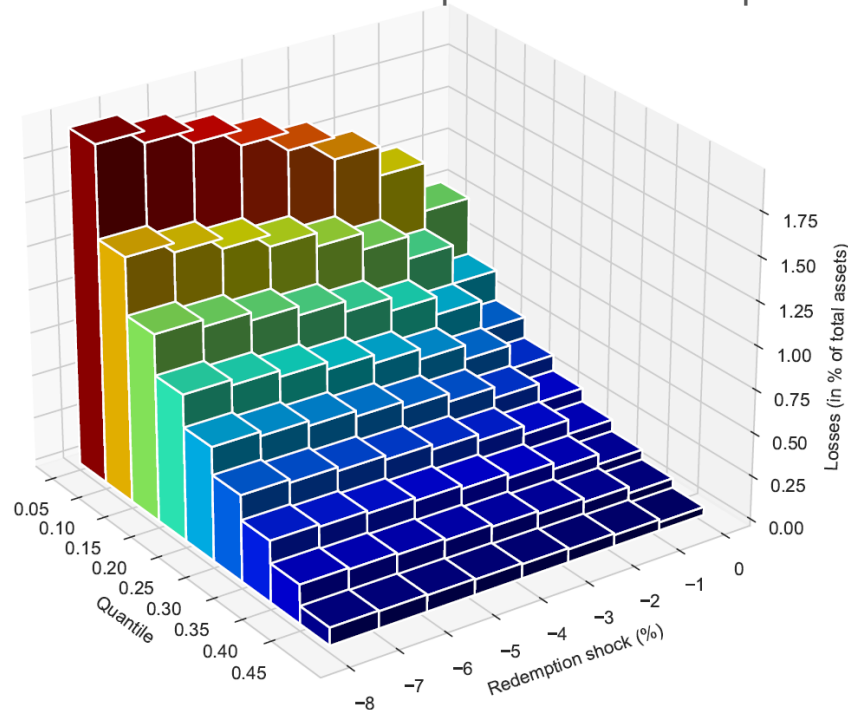


Assumed initial redemption shock for investment funds is -5%.

# Fire sale simulations (III)

- **Sensitivity analysis** shows a sub-linear increase in system-level losses with the increase of redemptions

Losses for different redemption shocks and quantiles



# 5



## Conclusion and next steps



# Conclusion

- We estimated **security-level price impact** parameters for different, arbitrary amounts sold
- **Price-at-risk** is a useful complement to standard ‘average’ price impact parameters used in the literature
- Taking into account the **heterogeneity** across securities alleviates some of the risks shown by fire sale models that apply **homogenous** price impact parameters

# Next steps

- Increase security **coverage**
- Introduce **intraday** calibration
- Improve **direction** identification
- **ESMA CCP stress test 2021/2022**: use of our estimates as top-down benchmark to validate CCP bottom-up results
- **Link financial stability and monetary policy**: evaluate possible changes in bond yields at the tail conditional on changes in ECB asset purchases

Thank you!



# Why we care?

## Literature

- Price Impact
  - Gabaix et al., 2003
  - Bouchaud, 2017
  - Patzalt and Bouchaud, 2018
  
- Contagion modelling
  - Shleifer and Vishny, 2011
  - Cont and Schaanning, 2017
  
- Quantile regression
  - Koenker and Hallock, 2001
  - Adrian and Brunnermeier, 2011

# Recap assumptions

Based on these assumptions previously introduced, and the literature, we expect that:

- Trades move prices
- Impact is permanent
- Existence of correlation between trade direction and price impact
- Price impact is concave

# How to model price impact?

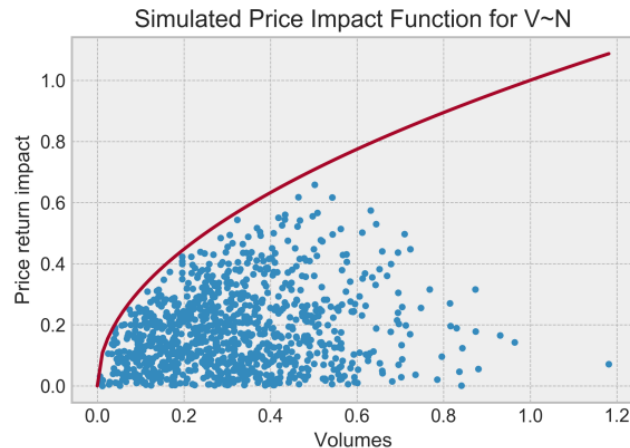
## Drawback of square-root functional form for non-metaorder data

Price impact diverge as the volume size increases.

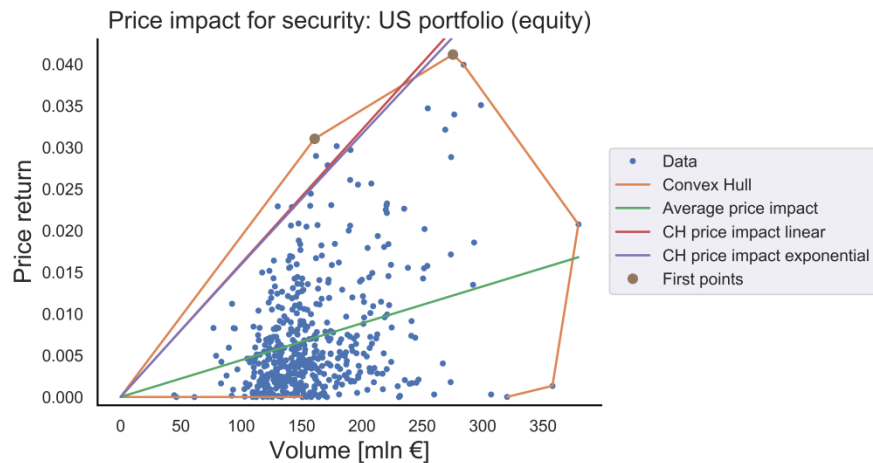
Which implies:

$$R(T, v) > 100\%$$

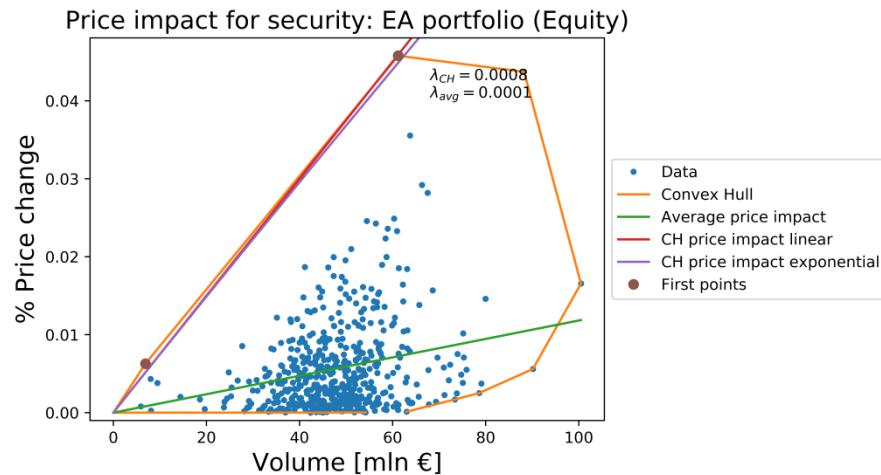
Allowing the price to drop below 0



# Empirical



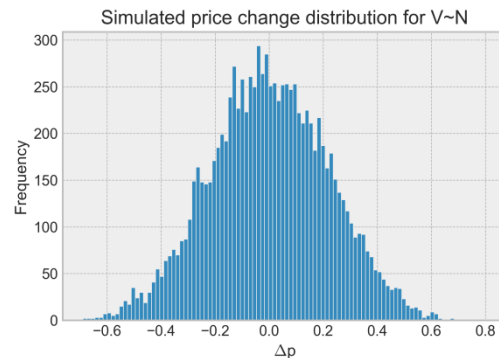
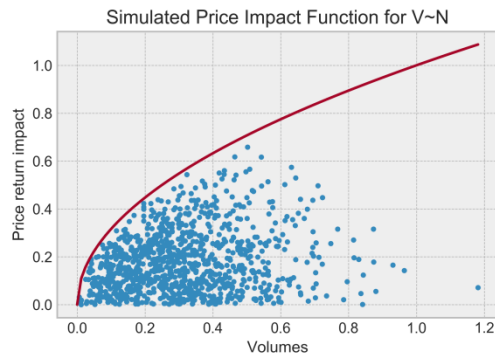
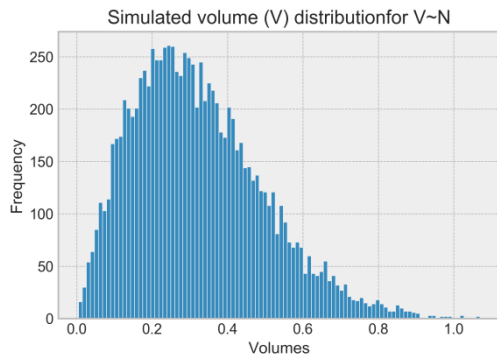
Price impacts on US portfolio



Price impacts on EA portfolio

# Simulation: Normal distribution

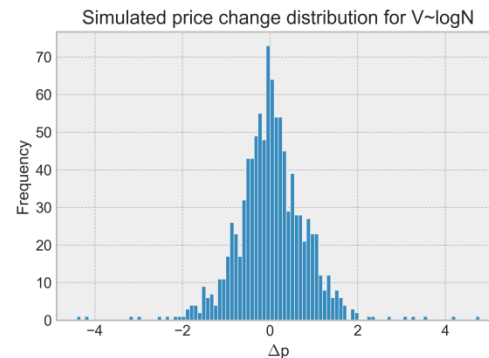
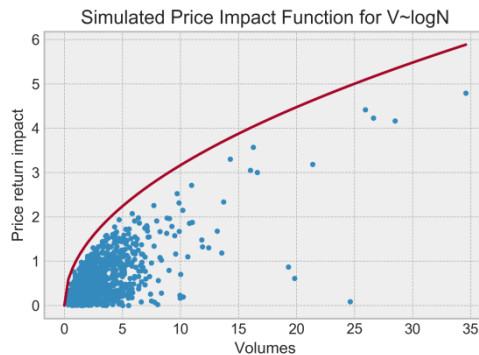
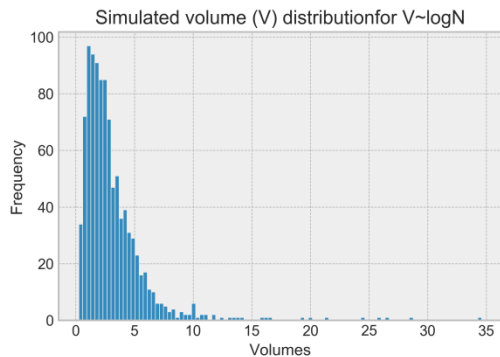
- Normal





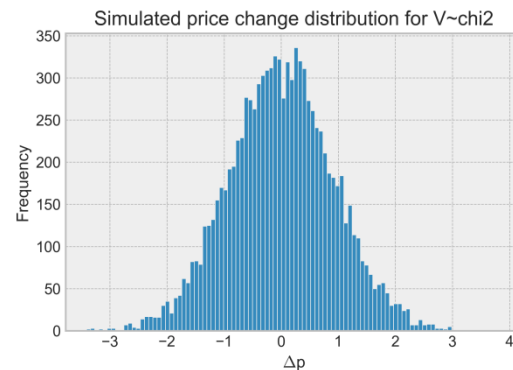
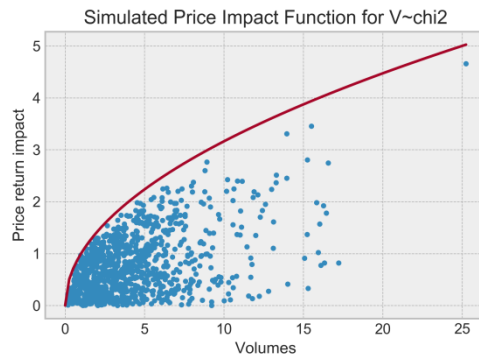
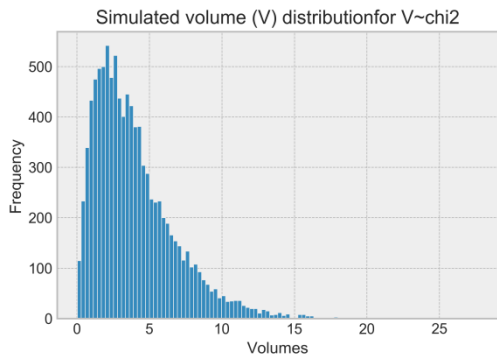
# Simulation: Log Normal distribution

- Log normal



# Simulation: Chi squared distribution

- Chi squared

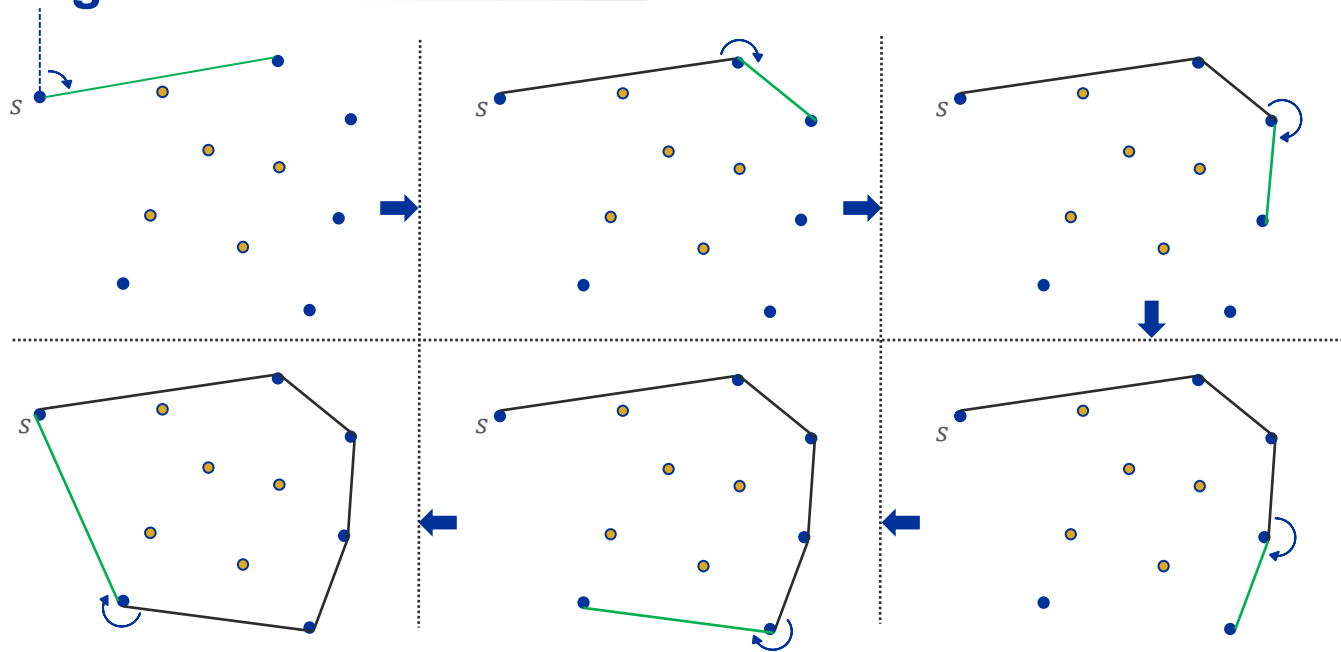


# Computing the Convex Hull:

## Jarvis' algorithm

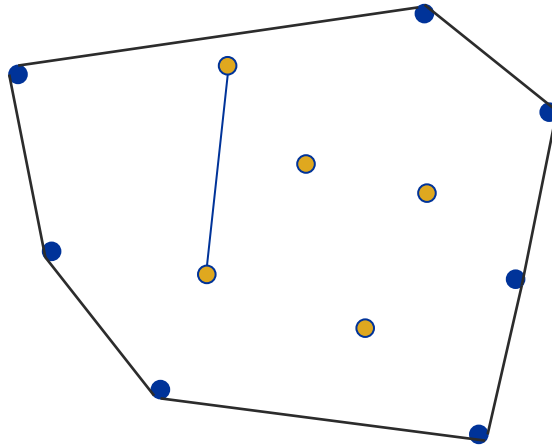
**Convex Hull (CH)** of a set of points is the smallest convex subset that contains the points. One way of imagining a Convex Hull is by stretching a rubber band around all point in a set.

# Computing the Convex Hull: Jarvis' algorithm



Convex hull Jarvis' algorithm

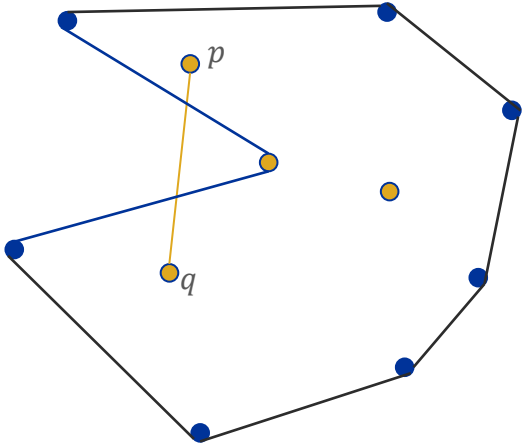
# Computing the Convex Hull: Jarvis' algorithm



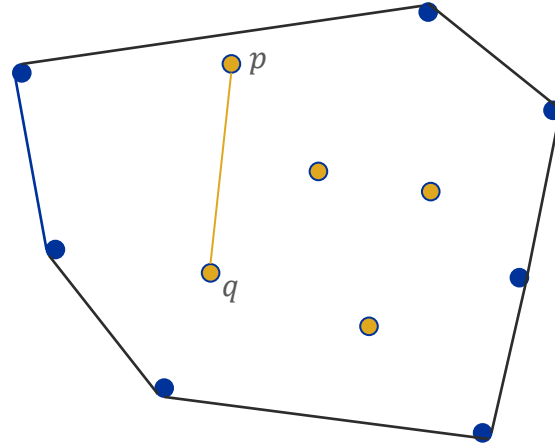
Set of points and its convex hull

Convex hull vertices and edges are blue and black; interior points are yellow

# Computing the Convex Hull: Jarvis' algorithm

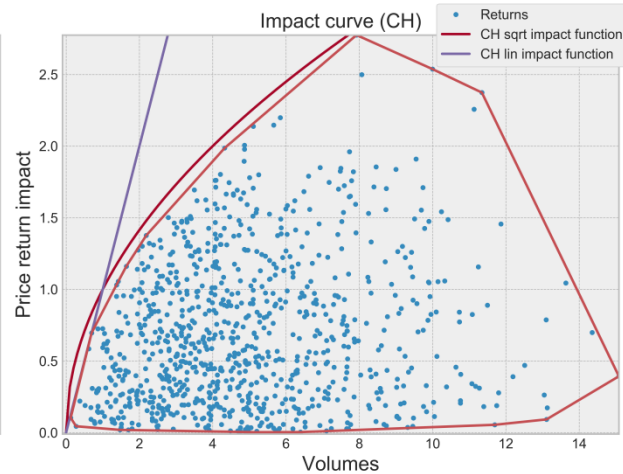
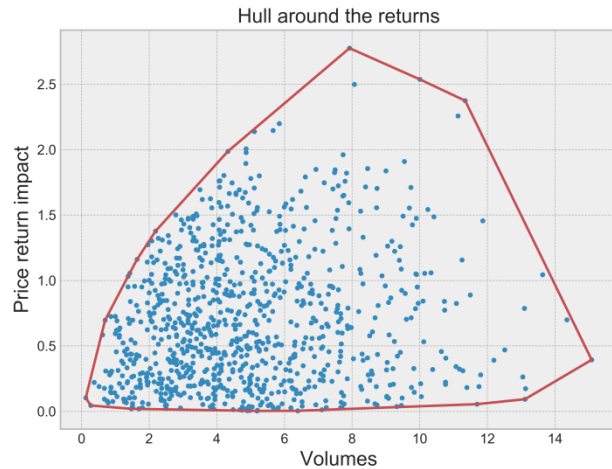


Not convex

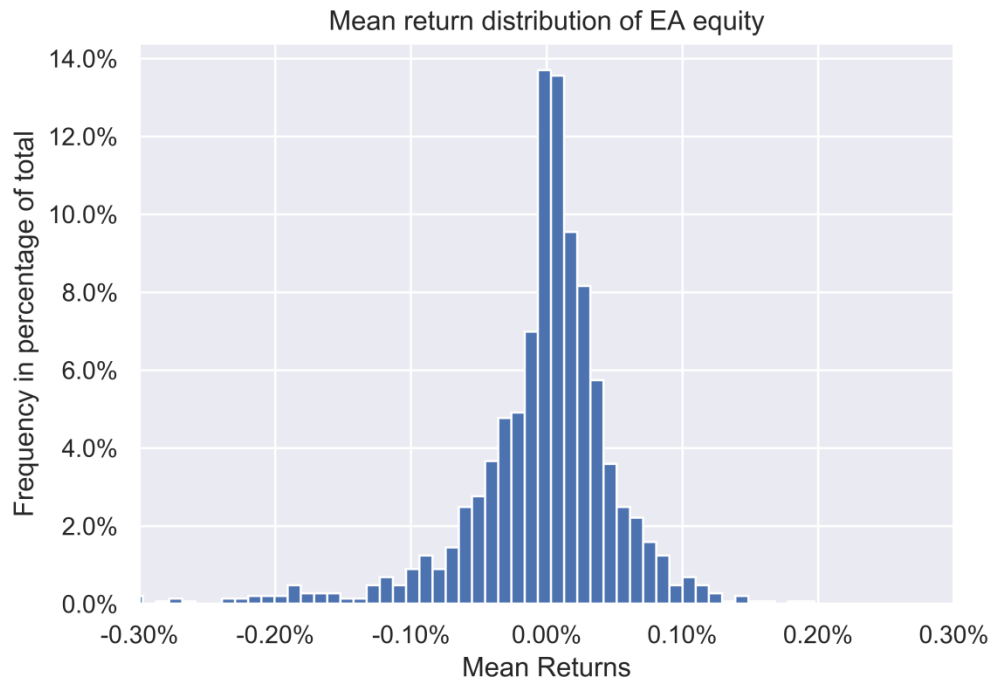


Convex

# Impact curve using the Convex Hull



# Distributions of the means of returns





# Data coverage

Market cap.	Number of equities		Outstanding amount (EUR Bn)	
	FC	NFC	FC	NFC
1b	298	488	123	198
10b	145	330	465	1379
100b	33	110	971	3514
Total	476	928	1560	5090

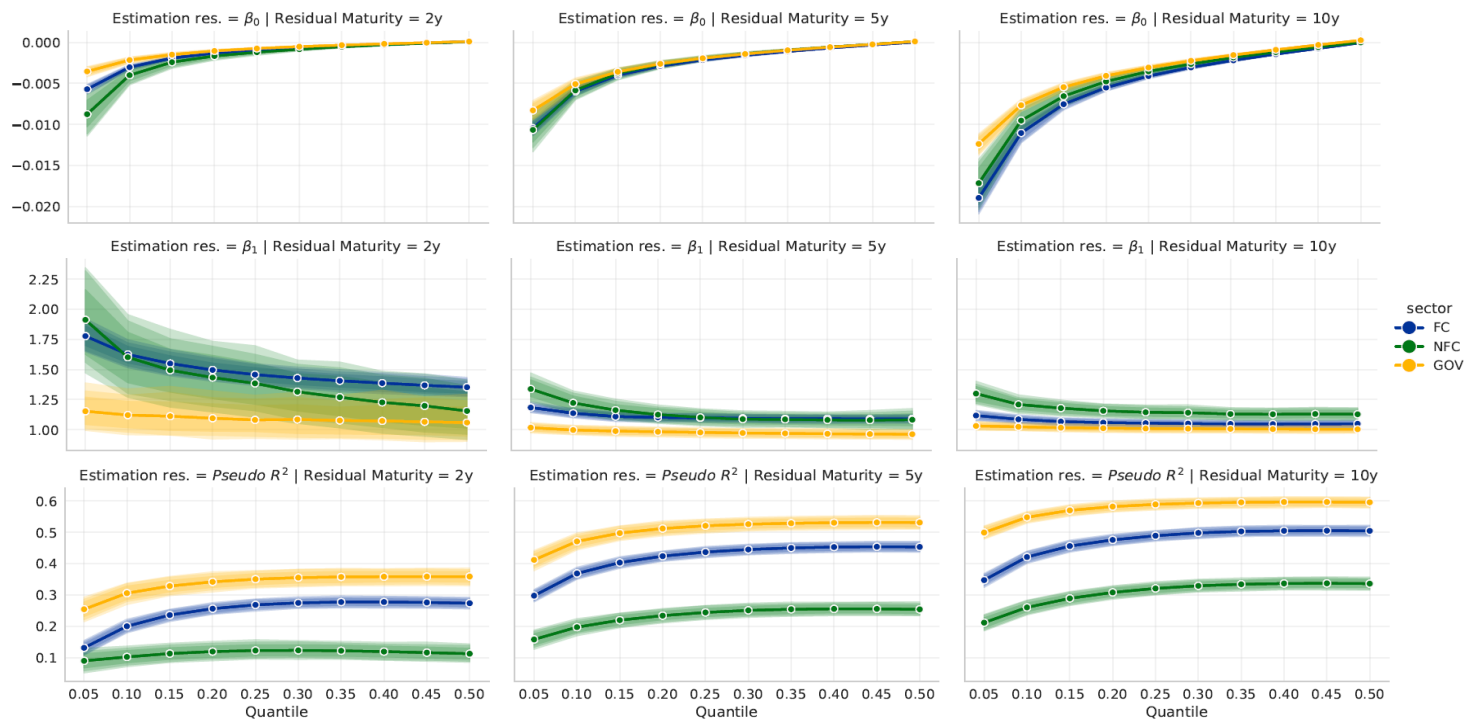
Source: Refinitiv (Eikon)

Country (ISO code)	Number of bonds			Outstanding amount (EUR Bn)		
	FC	GOV	NFC	FC	GOV	NFC
AT	67	17	27	39	173	17
BE	32	34	28	24	247	20
DE	486	433	66	474	1381	48
ES	139	136	19	141	907	12
FR	318	133	200	357	1533	158
IT	166	143	70	106	1616	48
NL	318	24	39	280	276	25
Total	1526	920	449	1421	6132	327

Source: Market Data Provision (MDP), Bloomberg

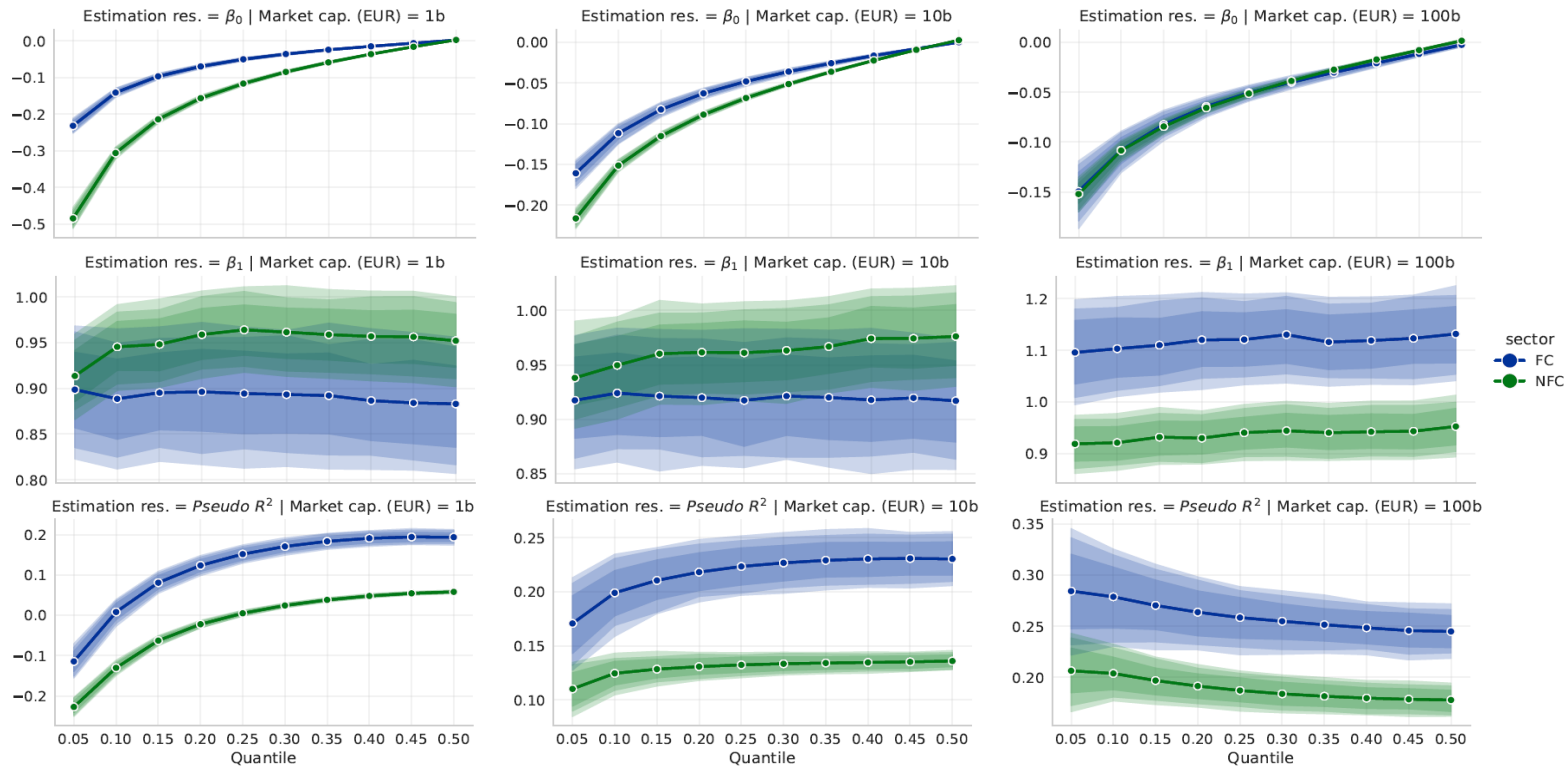
# Estimation results

## Bonds



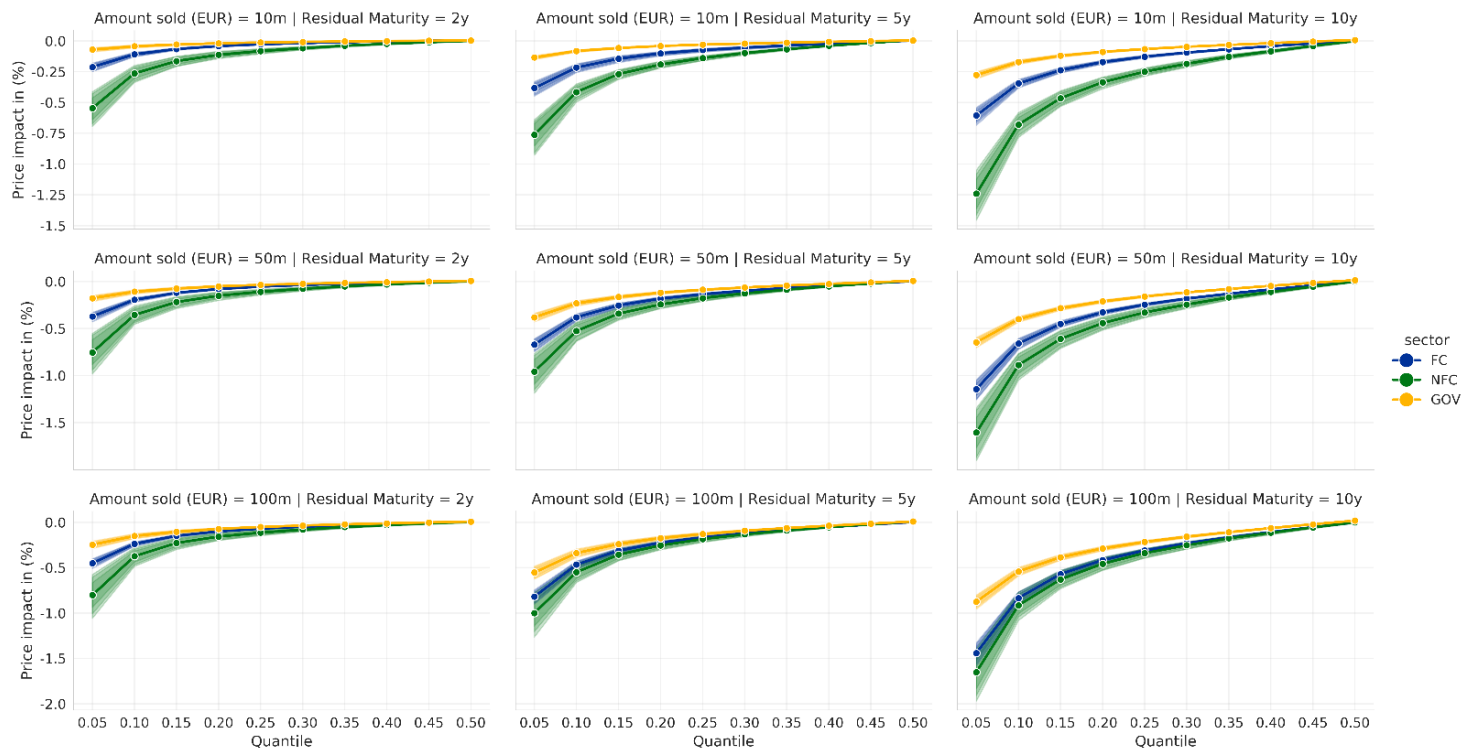
# Estimation results

## Equities



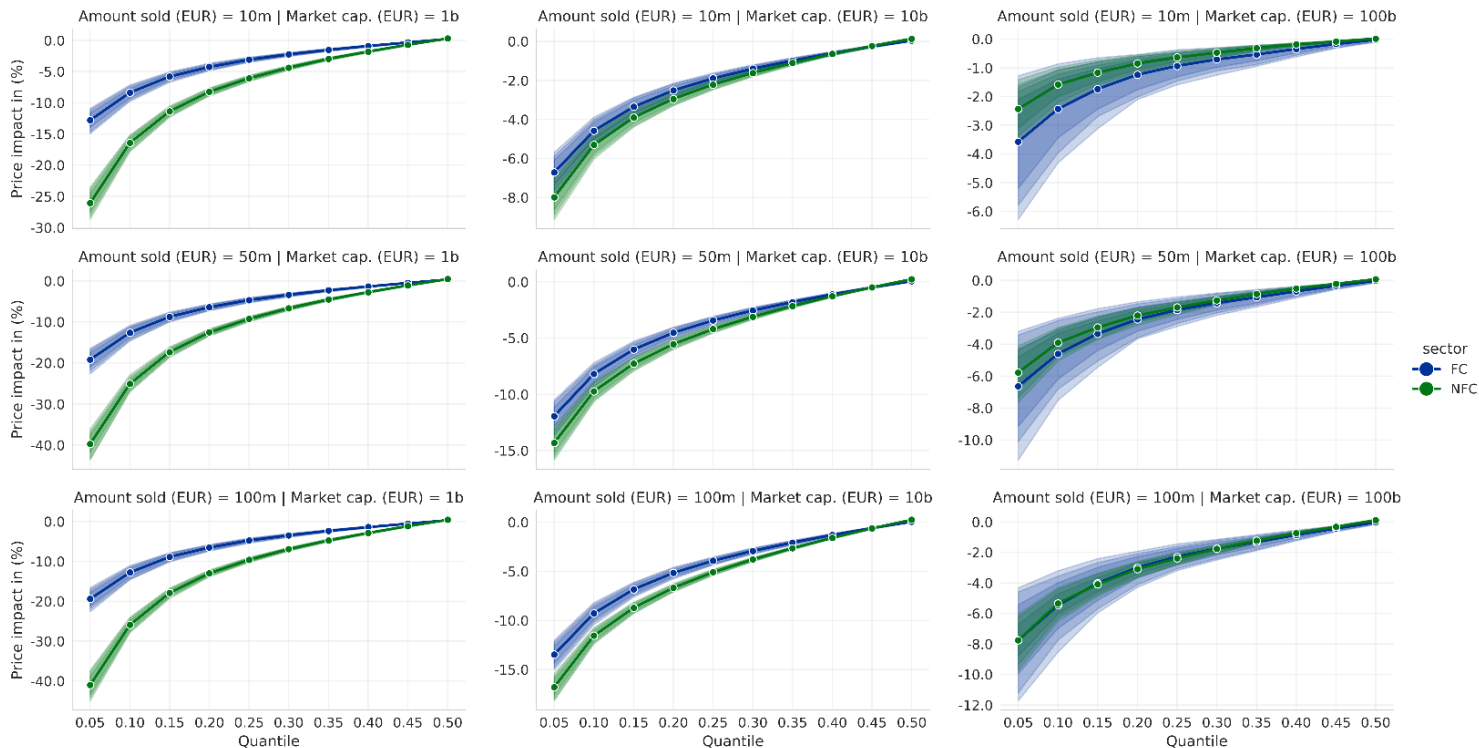
# Security level impact size

## Bonds



# Security level impact size

## Equities



# Bond level impact size

