A Macro-Finance model with Realistic Crisis Dynamics

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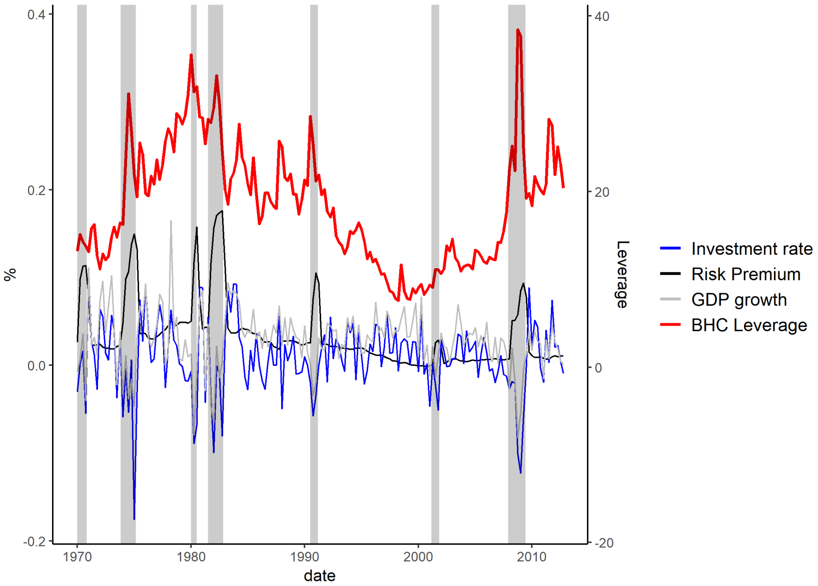
0.4
0.2
0.0
-0.2
%
40
20
0
-20
Leverage
1970
1980
1990
2000
2010
date
Investment rate
Risk Premium
GDP growth
BHC Leverage

Bank Failures

- Around 297 bank failures in 2008-2010 (**12 times** more than pre-crisis period)
- By default volume, 80% of the Moody's issuer default in 2009 came from Financial Institutions

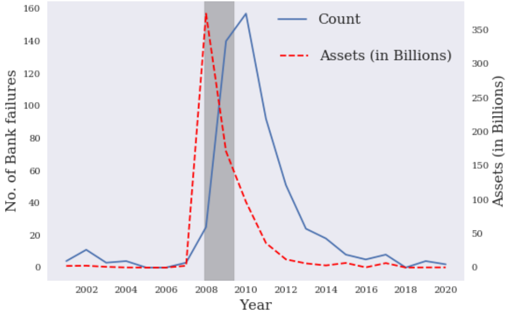

Figure: Bank failures from 2001 till 2020. Source: Federal Deposit Insurance Corporation.

Balance sheet recessions

Goes back to Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999).
Aggregate shocks $\implies$ **weaker balance sheets** $\implies$ amplification and persistence

- Financial frictions create balance sheet channel
- How to quantitatively explain crisis dynamics?
- Today, we will see a model that jointly explains
  1. Time varying risk premium
  2. Output (GDP) dynamics
  3. Leverage patterns
  4. Sluggish crisis recovery

This paper

1. Macro-finance model with financial amplification to explain deep and persistent financial crises
   1. Two sector model with households, and experts facing a) stochastic productivity and b) regime-dependent exit rate
   2. Multi-dimensional model → solved using active deep learning that encodes economic information as regularizers (Gopalakrishna (2021))
2. Quantitative Analysis
   1. Two key trade-offs in benchmark model with constant productivity and no exit
      (a) unconditional risk premium and probability of crisis,
      (b) conditional risk premium (amplification) and duration of crisis (persistence)
   2. My model resolves these tensions and provides a better match to data

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Literature (partial list)

- **Financial Frictions:** Bernanke et al (1999), Kiyotaki and Moore (1997), Bernanke and Gertler (1989), Gertler and Kiyotaki (2015)
- **Global methods:** He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014, 2016), DiTella (2016), Kurlat (2018), Adrian and Boyarchenko (2014)
- **Solution technique:** Duarte (2017), Fernandez-Villaverde et al (2020), Brunnermeier and Sannikov (2016)
- **Quantification:** He and Krishnamurthy (2019), Krishnamurthy and Li (2020)

Economic Mechanisms

Setup:

- Two classes of agents: **Households**, and **Experts** (financially constrained, leveraged).
- Normal times: More productive experts sufficiently capitalized, hold all capital

Crisis dynamics:

- **Capital and Productivity shock**: negative shock $\rightarrow \downarrow$ leveraged expert net worth $\rightarrow$ amplification (large risk premium, GDP falls, investment falters, and return volatility increases)
- **Regime-dependent exit**
  1. Larger exit in crisis pushes economy deeper into recession
  2. only way to come out of crisis is by increased expert productivity. Slow mean reversion in productivity $\implies$ delayed recovery (persistence)

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Model

- Experts borrow from households through risk-free debt, invest in risky capital
- Both experts and households can hold capital, but experts get higher return
- **Friction**: Skin-in-the game constraint
- OLG (Garleanu and Panageas, 2015)

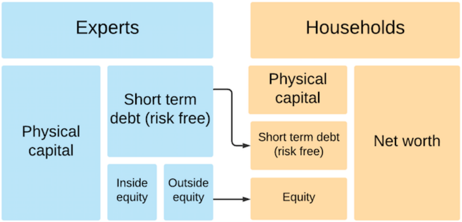

Figure: Balance sheet of households and experts

Model

AK technology $y_{j,t} = a_{j,t} k_t, \quad j \in e, h$

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta)dt + \sigma dZ_t^k$$

1. Productivity of experts is time-varying and follows the process

$$da_{e,t} = \pi(\hat{a}_e - a_{e,t})dt + \nu(\overline{a}_e - a_{e,t})(a_{e,t} - \underline{a}_e)dZ_t^a$$

   with $d\langle Z_t^k, Z_t^a \rangle = \varphi dt > 0$ and $a_h < \underline{a}_e < \hat{a}_e < \overline{a}_e$
   → Reflects bank economies of scale

2. Experts exit at rate $\tau_t \in \{\tau_{normal}, \tau_{crisis}\}$, with $\tau_{crisis} = \mathbf{9} \times \tau_{normal}$.
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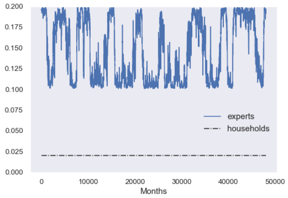

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Model

- Preferences: Stochastic differential utility (risk aversion γ, and IES=1)

$$U_{j,t} = E_t\left[\int_t^\infty f(c_{j,s}, U_{j,s})ds\right]$$

with

$$f(c_{j,t}, U_{j,t}) = (1-\gamma)\rho U_{j,t}\left(\log(c_{j,t}) - \frac{1}{1-\gamma}\log\Big((1-\gamma)U_{j,t}\Big)\right)$$

- Agents maximize lifetime utility subjected to wealth constraints
- Experts take exit rate into account in their optimization problem

Model

Experts solve

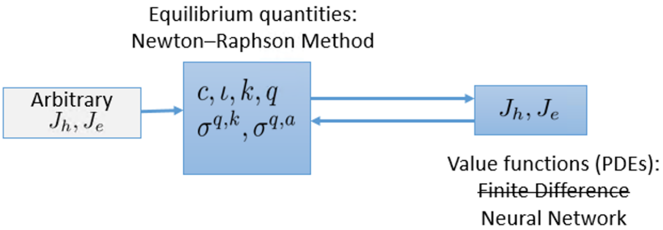
$$U_{e,t} = \sup_{C_{e,t}, K_{e,t}, \chi_{e,t}} E_t\left[\int_t^{\tau'} f(C_{e,s}, U_{e,s})ds + U_{h,\tau'}\right]$$

$$\text{s.t.} \quad \frac{dW_{e,t}}{W_{e,t}} = \left(r_t - \frac{C_{e,t}}{W_{e,t}} + \frac{q_t K_{e,t}}{W_{e,t}}(\mu^R_{e,t} - r_t - (1-\chi_{e,t})\epsilon_{h,t}) - \lambda_d + \frac{\bar{z}}{z_t}\lambda_d - \tau_t\right)dt$$
$$+ \sigma_{w_e,t}\left((\sigma + \sigma^{q,k}_t)dZ^k_t + \sigma^{q,a}_t dZ^a_t\right)$$

- Transition time τ' is exponentially distributed with rate $\tau_t \in \{\tau_{normal}, \tau_{crisis}\}$
- $\frac{q_t K_{e,t}}{W_{e,t}}$: fraction of capital invested
- $\chi_{e,t}$: fraction of equity retained in balance sheet

Solution technique: Markov equilibrium

- Two state variables: wealth share of experts z_t (endogenous), productivity of experts $a_{e,t}$(*exogenous*)
- Solution boils down to solving coupled system of PDEs in J_h and J_e



- Neural network approach (ALIENs) developed in Gopalakrishna (2021)

Capital Price

Capital Price

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Benchmark model (constant productivity, no exit)

- Variables of interest: risk premium, probability of crisis, duration of crisis. Two key tensions in the model
  1. Unconditional risk premium vs Probability of crisis
  2. Conditional risk premium (amplification) vs Duration of crisis (persistence)
- Crisis moments: Reinhart and Rogoff (2009), and NBER.
- Risk premium: Estimate from S&P500 (1945-2018) returns

$$R^e_{t+1} = a + \beta * D_t/P_t + \beta_{rec} * \underbrace{1_{Rec}}_{\text{recession dummy}} * D_t/P_t + \beta_{fin} * \underbrace{1_{fin}}_{\text{financial crisis dummy}} * D_t/P_t + \epsilon_t$$

	Data			Benchmark Model RA=1		Benchmark Model (RA=20)	
	All	Recession	Crisis	All	Crisis	All	Crisis
E(Risk premium)	7.5	16.6	25.0	1.7	13.4	7.3	-
Std(Risk premium)	5.1	6.5	7.4	2.8	1.3	0	-
Probability of Crisis	7			7.8		0	

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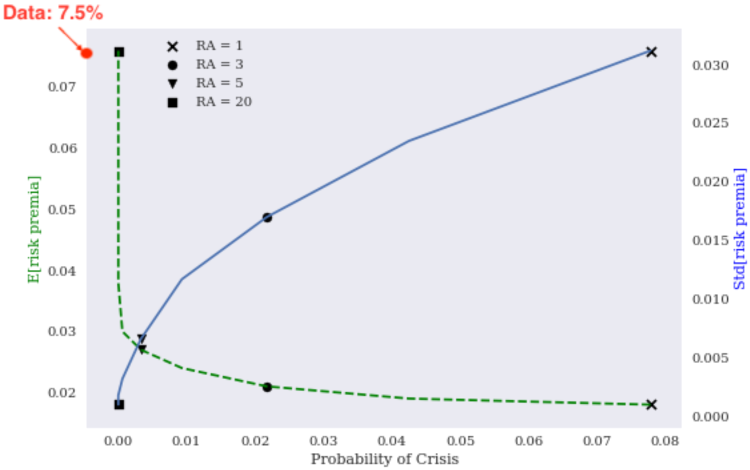
Trade-off 1: Risk premium and Prob. of crisis



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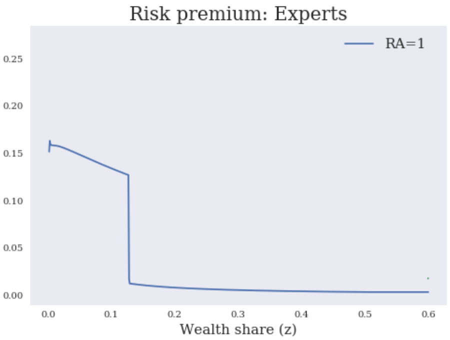

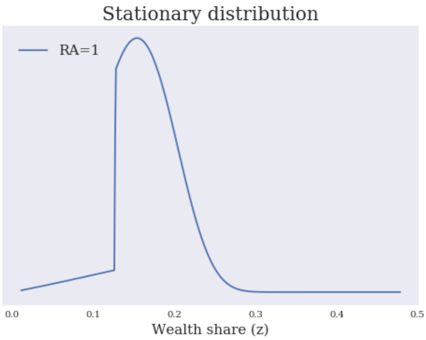

	Unconditional Risk premium (%)	Probability of Crisis (%)
RA=1	1.7	7.8
Data	7.5	7

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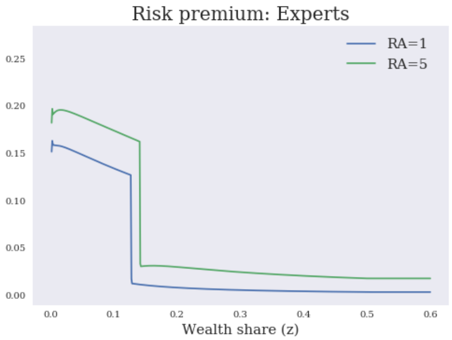

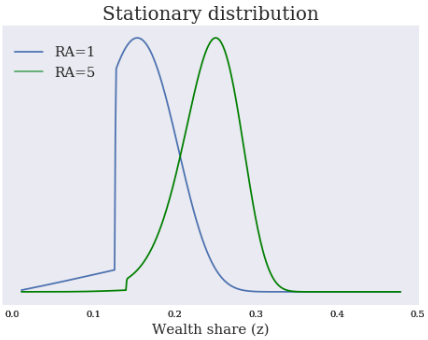

	Unconditional Risk premium (%)	Probability of Crisis (%)
RA=1	1.7	7.8
RA=5	2.7	0.1
Data	7.5	7

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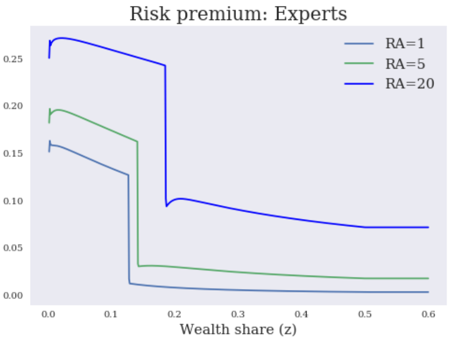

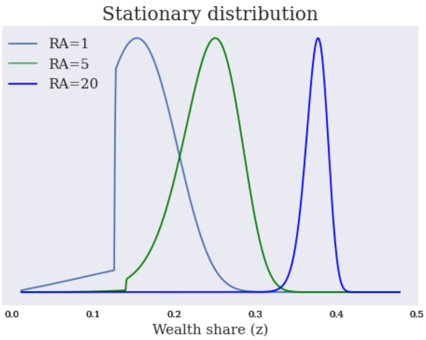

	Unconditional Risk premium (%)	Probability of Crisis (%)
RA=1	1.7	7.8
RA=5	2.7	0.1
RA=20	7.6	0
Data	7.5	7

Trade-off 2: Conditional risk premium (amplification) and duration (persistence)

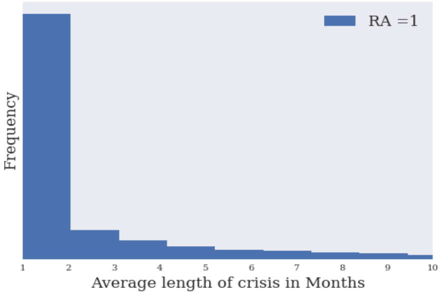

Figure: Model-implied average duration of crisis

- Average crisis duration
  1. **Empirical**: 17 months (Source: NBER)
  2. Model implied:

	Conditional Risk premium (%)	Duration of Crisis (months)
RA=1	13.4	5.7
Data	25	17.0

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	Conditional Risk premium (%)	Duration of Crisis (months)
RA=1	13.4	5.7
RA=2	14.5	4.8
Data	25	17.0

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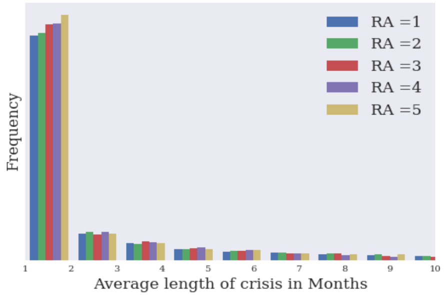

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RA=2	14.5	4.8
RA=3	15.1	4.2
RA=4	16.0	3.9
RA=5	16.6	3.3
Data	25	17.0

- Higher RA → higher conditional risk premium → experts build wealth faster and move out of crisis quickly
- The risk premium effect dominates the capital price and investment effect

Other channels that generate such tension

Risk aversion is one among other channels that generate the tension

- **Skin-in-the-game constraint**: Tighter financial constraint leads to amplified crisis, at the cost of reduced persistence
- **Exogenous volatility**: Higher volatility has similar effects
- Not a matter of calibration: auxiliary features that cause high financial amplification mechanically induce faster recovery through higher conditional risk premium

Benchmark model evaluation

	Quantity of interest	Success level	Comments
Macroeconomic	GDP/Output growth	High	✓
	Investment rate	Low	Low variation
Intermediary	Leverage	High	✓
	Cyclicality of leverage	High	✓
Crises	Probability of crises	Moderate	Attenuates avg risk premium
	Duration of crises	Low	Attenuates amplification
Asset price	Conditional risk premium	High	✓
	Unconditional risk premium	Low	Cannot match prob. of crisis
	Std. of risk premium	Moderate	-
	Conditional volatility	High	✓
	Unconditional volatility	Low	Shiller puzzle

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Comparison

Fix probability of crisis at 7% and evaluate moments.

	My model			Benchmark model		
	All	Crisis	Normal	All	Crisis	Normal
E[leverage]	2.80	4.79	2.62	3.23	5.50	3.10
E[inv. rate]	7.70%	2.80%	8.20%	6.00%	5.00%	6.00%
E[risk free rate]	0.90%	-7.20%	1.70%	4.80%	0.00%	5.00%
E[risk premia]	6.70%	17.50%	5.70%	1.70%	13.40%	1.00%
E[GDP growth rate]	1.20%	-8.00%	1.90%	2.30%	-7.90%	2.70%
Std[inv. rate]	3.18%	1.31%	2.91%	0.36%	1.09%	0.11%
Std[risk premia]	5.35%	1.57%	4.45%	2.82%	1.31%	0.18%
Std[risk free rate]	3.98%	1.64%	3.21%	1.19%	0.42%	0.28%
Corr(leverage,shock)	-0.25	-0.17	-0.30	-0.28	-0.05	-0.25
Probability of crisis	7.0%			7.80%		
Duration of crisis (months)	18.5			6		

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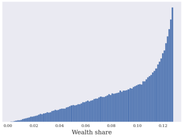
How are the tensions resolved?

- **Benchmark**: Only one shock: i.i.d Brownian.
  1. In steady state, capital shock to risk averse experts is not enough to generate sufficient crises periods (trade-off 1 ✗)
  2. Once in crisis, amplification happens but experts repair their balance sheet faster $\implies$ quick recovery (trade-off 2 ✗)
- **My model**: Two correlated Brownian shocks plus higher exit in crisis.
  1. In steady state, capital shock to risk averse experts also lowers productivity and generates crisis (trade-off 1 ✓)
  2. Once in crisis, amplification happens but experts exit economy at higher rate
  3. Productivity shoots up slowly $\implies$ sluggish recovery (trade-off 2 ✓)



(a) Benchmark model: left tail of distribution

(b) My model: left tail of distribution

Conclusion

- Wealth share of intermediaries alone cannot jointly match asset pricing, output, and crisis moments
  1. Trade-off between unconditional risk premium and probability of crisis
  2. Trade-off between conditional risk premium (amplification) and duration of crisis (persistence)
- A model of stochastic productivity and regime-dependent exit generates realistic crisis dynamics, and a better match to data
- Active machine learning opens new avenues for future research
  1. 'Brunnermeier-Sannikov meets Bansal-Yaron' economy (Gopalakrishna (2021))
  2. Heterogeneous intermediaries
  3. Main street vs Wall street disconnect, good booms vs bad booms
  4. Sunspot equilibria
  5. ....and more

Thank you!